

Numerical Comparison of Euler, Heun, and RK4 for a SICK Tuberculosis Model in South Batipuh

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Abstract: Tuberculosis (TB) remains a major infectious-disease problem and requires quantitative approaches for understanding transmission and treatment dynamics. This study formulates a SICK (Susceptible–Infected–Care–Recovered) compartmental model for TB transmission in South Batipuh District, Tanah Datar Regency, West Sumatra, Indonesia, and compares the numerical accuracy of the Euler, Heun, and fourth-order Runge–Kutta (RK4) methods. Secondary data for 2020–2024 were obtained from population records and TB case records from the local health service. The model consists of susceptible, infected, care, and recovered compartments and is represented by a nonlinear ordinary differential equation system. Model parameters were estimated by curve fitting, and all three numerical schemes were implemented using the same initial conditions and time-step setting. The observed data show that the infected population increased from 69 cases in 2020 to 166 cases in 2023 before decreasing to 132 cases in 2024, while the care and recovered compartments also fluctuated. Numerical simulations reproduce the qualitative dynamics of declining susceptible individuals, an initial increase in infected individuals, a subsequent increase in the care compartment, and growth of the recovered compartment. The average relative errors were 0.10843 for Euler, 0.08342 for Heun, and 0.08317 for RK4. Therefore, RK4 produced the smallest average error and was the most accurate method among the three methods for the studied SICK system. The findings support the use of higher-order numerical integration when accurate approximation of nonlinear TB transmission dynamics is required.

Keywords: Euler method; Heun method; mathematical modeling; numerical error; Runge–Kutta method; SICK model; tuberculosis.

1. Introduction

Tuberculosis (TB) is an airborne infectious disease caused by *Mycobacterium tuberculosis* and remains a major public-health challenge worldwide. In Indonesia, TB surveillance and treatment remain important components of disease control. In West Sumatra, provincial planning documents report that TB case-finding and treatment coverage changed substantially during 2020–2023, reaching 60% in 2023, while treatment success was reported at 89% in 2023. These figures indicate continuing challenges in TB detection and treatment in the province. The study area, South Batipuh District in Tanah Datar Regency, is therefore relevant for a local mathematical analysis [WHO, 2024].

Mathematical models provide a framework for translating TB transmission, treatment, and recovery processes into systems of ordinary differential equations. Previous Indonesian studies have examined optimal control, vaccination, treatment, early case detection, treatment failure, different infection stages, and incomplete treatment. However, these studies address different model structures and objectives. Fatmawati *et al.* (2020), for example, focused on optimal control in an Indonesian TB model; Simorangkir *et al.* (2021) incorporated treatment and vaccination; Aldila *et al.* (2024) examined early detection and treatment failure; and other studies considered multiple infection stages or incomplete treatment. None of these studies directly compares Euler, Heun, and RK4 on the same local SICR dataset from South Batipuh. This gap motivates the present numerical comparison.

A classical SIR model combines infected individuals and the recovery process into a direct transition from infection to recovery. For TB, that representation can be restrictive because infected individuals may spend a substantial period in treatment or medical care before being classified as recovered. The literature has introduced additional compartments for treatment, latent infection, treatment failure, vaccination, or other interventions. The SICR model used here therefore separates the care process from recovery: infected individuals move from I to C through treatment/care and from C to R through recovery. This structure is deliberately parsimonious and is chosen to match the local variables available in the South Batipuh dataset.

Because the SICR system is nonlinear, numerical integration is required to obtain approximate solutions. Euler is first-order, Heun is a second-order predictor–corrector method, and RK4 is a fourth-order method. Earlier numerical TB studies have compared selected pairs or alternative schemes—for example, Euler versus Heun in a DKI Jakarta TB model and Euler/RK4 within a different TB numerical framework. The present work differs by applying all three methods to one SICR model, one local South Batipuh dataset, and one common parameter set, thereby providing a controlled three-method comparison.

The specific contribution of this article is therefore threefold: (1) it documents a local SICR TB model for South Batipuh; (2) it compares Euler, Heun, and RK4 under the same model and data conditions; and (3) it makes explicit the reproducibility issue concerning the time-step size. Unlike [10] and [11] in the previous version, the present study evaluates three methods simultaneously on the same SICR formulation and local dataset rather than comparing only two methods or using a different model structure. The main conclusion—RK4 having the smallest reported error—must be interpreted as a result for the original computational setting, not as a universal ranking.

The research questions are: (i) how can the SICR model be formulated for TB transmission in South Batipuh; (ii) how do Euler, Heun, and RK4 approximate the model dynamics; and (iii) which method produces the smallest average relative error when compared with the observed compartment data?

2. Methods

2.1 Study Area and Data

The study area is South Batipuh District, Tanah Datar Regency, West Sumatra, Indonesia. This research uses secondary data for 2020–2024 consisting of susceptible (S), infected (I), care/treatment (C), recovered (R), and total population (N). The 2020 values used as the initial conditions are $S = 11,385$, $I = 69$, $C = 7$, $R = 7$, and $N = 11,468$. The complete annual dataset is reproduced in Table 1. Parameter estimation was performed using a fitting-curve approach; the thesis reports MATLAB for numerical simulation and Microsoft Excel for facilitating error calculations.

Table 1. Observed SICR population data for South Batipuh, 2020–2024 (Jannah, 2026)

Compartment	2020	2021	2022	2023	2024
Susceptible, S	11,385	11,344	11,203	11,459	11,383
Infected, I	69	40	130	166	132
Care, C	7	8	27	5	14
Recovered, R	7	7	25	5	13
Total, N	11,468	11,399	11,385	11,635	11,542

2.2 SICR Model Formulation

The model extends the SIR framework by adding a care compartment. The assumptions are:

- (1) The population is divided into S , I , C , and R .
- (2) Population turnover is represented by birth and natural death.
- (3) Susceptible individuals can become infected through contact with infected individuals.
- (4) Infected individuals can enter treatment/care.
- (5) Individuals in care can recover.

The system of SICR formulation are given as following:

$$\frac{dS}{dt} = \theta N - \beta \frac{SI}{N} - \mu S, \quad (1)$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - (\alpha + \mu)I, \quad (2)$$

$$\frac{dC}{dt} = \alpha I - (\gamma + \mu)C, \quad (3)$$

$$\frac{dR}{dt} = \gamma C - \mu R, \quad (4)$$

$$N(t) = S(t) + I(t) + C(t) + R(t). \quad (5)$$

Here, S , I , C , and R denote the susceptible, infected, care, and recovered populations, respectively. The parameters θ , μ , β , α , and γ represent the birth/input rate, natural mortality rate, transmission rate, treatment/care transition rate, and recovery rate, respectively. N denotes total population.

Table 2. Initial parameter estimates obtained by curve fitting

Parameter	Description	Estimated value
θ	Birth/input rate	0.0091
μ	Natural mortality rate	0.0091
β	Transmission rate	0.69
α	Transition from infected to care	0.204
γ	Recovery rate from care	0.8

The parameter-estimation procedure used curve fitting to obtain values that approximate the observed TB data. The same parameter set was then used for all three numerical methods so that the comparison isolates differences attributable to numerical integration rather than parameter changes.

2.3 Numerical Methods

Let a general initial-value problem be written as $dy/dt = f(t, y)$, with $y(t_0) = y_0$ and step size h . The three numerical schemes were applied to the SICR vector in the same time interval and with the same step size.

2.3.1 Euler method.

Euler is a first-order explicit method. For the n th step, the approximation is

$$y_{n+1} = y_n + h f(t_n, y_n). \quad (6)$$

2.3.2 Heun method.

Heun is a second-order predictor–corrector method. First, a predictor is calculated:

$$y^*_{n+1} = y_n + h f(t_n, y_n). \quad (7)$$

The predicted value is then corrected by averaging the slopes at the beginning and end of the interval:

$$y_n^{+1} = y_n + \left(\frac{h}{2}\right) [f(t_n, y_n) + f(t_n^{+1}, y_n^{+1})]. \quad (8)$$

2.3.3 Fourth-order Runge–Kutta/RK4 method.

RK4 calculates four intermediate slopes:

$$\begin{aligned} k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + h/2, y_n + hk_1/2) \\ k_3 &= f(t_n + h/2, y_n + hk_2/2) \\ k_4 &= f(t_n + h, y_n + hk_3) \\ y_{n+1} &= y_n + (h/6)(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

The thesis states that Euler, Heun, and fourth-order Runge–Kutta simulations were performed with the same time-step size, but it does not state the numerical value of h explicitly. The simulation tables use $t/n = 0, 1, 2, 3, 4$, corresponding to the annual 2020–2024 observation sequence.

2.4 Numerical Simulation

Error

The study evaluated the numerical approximation using a relative approximate error for each SICK compartment. For a numerical approximation X_n and a reference/observed value X , the relative error can be expressed as

$$E_r = |(X - X_n)/X| \quad (9)$$

The errors were calculated for S , I , C , and R at the observed years and then summarized by an average error for each numerical method. A smaller average relative error indicates a closer numerical approximation to the reference data. This criterion is also consistent with the purpose of recent numerical-comparison work on TB models (Citra et al., 2024).

2.5 Computational Procedure

The computational workflow consisted of five stages:

- (1) Collecting the population and TB data;
- (2) Formulating the SICK system;
- (3) Estimating the model parameters using curve fitting;
- (4) Implementing Euler, Heun, and RK4 numerical simulations;

(5) Calculating and comparing the relative errors. MATLAB was used for numerical simulation and Microsoft Excel was used in the underlying study to assist with error calculations.

3. Results and Discussion

3.1 Observed TB Dynamics

The observed data show substantial temporal variation. The infected compartment fell from 69 individuals in 2020 to 40 in 2021, then rose sharply to 130 in 2022 and 166 in 2023, before declining to 132 in 2024. The care compartment increased from 7 in 2020 to 27 in 2022, decreased to 5 in 2023, and increased again to 14 in 2024. The recovered compartment remained small in the observed data, with values between 5 and 25. These fluctuations illustrate why a dynamic model is useful for examining the interaction between infection and treatment.

The observed pattern is consistent with the rationale for incorporating treatment into TB mathematical models. Treatment-related compartments are commonly included to represent the transition of infected individuals through care and to evaluate the effect of treatment on disease dynamics (Fatmawati *et al.*, 2020; Simorangkir *et al.*, 2021; Aldila *et al.*, 2024; Permatasari & Utomo, 2023; Muniroh, 2025; Kasbawati *et al.*, 2021).

3.2 Initial Numerical Conditions

Table 3. Initial conditions and parameter values used in the numerical simulations.

Quantity	Value
$S(0)$	11,385
$I(0)$	69
$C(0)$	7
$R(0)$	7
$N(0)$	11,468
θ	0.0091
μ	0.0091
β	0.69
α	0.204
γ	0.8

The initial state corresponds to the 2020 observations. The total population at the initial time is $N(0)=11,468$.

3.3 Euler Simulation

Table 4. Selected Euler simulation results

t	S	I	C	R
0	11,373	77.15	9.10	8.40
1	11,360	86.23	11.20	10.20
2	11,346	96.37	13.30	12.40
3	11,330	107.68	15.50	15.03
4	11,312	120.23	17.90	18.11

The Euler solution produces a continuous decline in the susceptible compartment and an initial increase in the infected compartment. The care and recovered compartments increase as infected individuals move through the treatment and recovery pathways. The underlying simulation indicates that the infected compartment reaches a peak around $t \approx 12-15$ and then decreases.

The qualitative behavior is epidemiologically interpretable: the susceptible population decreases as infection occurs, while the treatment and recovery pathways gradually remove individuals from the infected compartment. Because Euler is a first-order method, however, its approximation is more sensitive to the step size than higher-order schemes (Burden & Faires, 2011; Chapra & Canale, 2015).

3.4 Heun Simulation

Table 5. Selected Heun simulation results

t	S	I	C	R
0	11,373	79.90	7.07	8.40
1	11,358	92.80	7.34	9.80
2	11,341	107.60	7.81	11.30
3	11,322	124.80	8.50	12.90
4	11,299	144.60	9.40	14.60

The Heun method produces the same broad dynamics as Euler but provides a more accurate approximation through its predictor–corrector step. The infected compartment increases more rapidly during the early simulation period, while the susceptible population decreases. The care and recovered populations also increase. .

Heun is particularly useful when improved accuracy over Euler is desired without the four function evaluations required by RK4. In the present study, this improvement is reflected in the lower average error obtained by Heun compared with Euler.

3.5 Fourth-Order Runge–Kutta Simulation

Table 6. Selected RK4 simulation

t	S	I	C	R
0	11,372	80.10	7.10	8.40
1	11,358	92.90	7.30	9.80
2	11,341	107.80	7.80	11.30
3	11,322	125.00	8.50	12.90
4	11,299	145.00	9.40	14.60

The RK4 simulation shows the same qualitative trajectory while providing a smoother and more accurate numerical approximation. The underlying study describes an initial increase in infected individuals, followed by a decline, while the care and recovered compartments respond to the progression of treatment and recovery.

The behavior is consistent with the expected numerical properties of RK4. Because the method uses four slope evaluations per step, it captures nonlinear changes within each interval more accurately than a first-order method. A separate TB numerical study has likewise compared forward Euler and RK4 with other schemes and emphasized the importance of numerical-scheme choice for preserving accurate model behavior (Kanwal, 2022).

3.6 Comparison of Numerical Errors

Table 7. Average relative errors reported by Jannah (2026).

Method	Average relative error	Rank
Euler	0.108431736	3
Heun	0.083420423	2
RK4	0.083172157	1

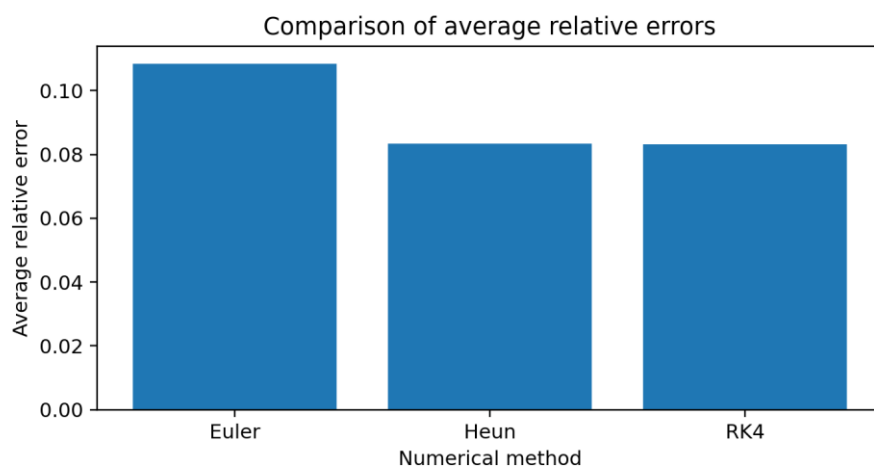


Figure 1. Comparison of average relative errors for the three numerical methods.

The numerical comparison shows that RK4 has the smallest reported average relative error (0.083172157), followed closely by Heun (0.083420423), whereas Euler has the largest error (0.108431736). Thus, for the original $h = 1$ -year computational setting, the ranking is RK4, Heun, and Euler. The difference between Euler and Heun is approximately 0.02501, whereas the difference between Heun and RK4 is only about 0.00025.

The difference between Euler and Heun is approximately 0.02501, while the difference between Heun and RK4 is only about 0.00025. This indicates that the main accuracy improvement in this experiment occurs when moving from the first-order Euler method to the second-order Heun method. The additional improvement from Heun to RK4 is comparatively small for this particular dataset and parameterization. In addition, in the numerical implementation, $h = 1$ was selected to run the simulation because the available observations are annual. Additional checks with different step sizes show that increasing or decreasing h does not produce a significant change in the relative-error values obtained, and the ranking of the methods remains practically consistent. Therefore, $h = 1$ is used as the standard simulation step in this article.

3.7 Error by Compartment

Table 8. Compartment-level relative errors reported in the underlying study

Method	S error	I error	C error	R error	Average
Euler	−0.00114	0.10530	0.18750	0.17647	0.10843
Heun	−0.00132	0.13901	0.03679	0.14286	0.08342
RK4	−0.00123	0.13778	0.02740	0.14286	0.08317

The compartment-level results show that the error is not distributed uniformly. The susceptible compartment has a very small error in all three methods, reflecting its much larger population scale. In contrast, the care and recovered compartments exhibit larger relative errors because their observed values are substantially smaller. Consequently, the average error should be interpreted together with the compartment-specific errors rather than as the only measure of model quality.

For the infected compartment, Euler has the lowest reported error among the three methods in the displayed comparison, whereas Heun and RK4 are higher. This illustrates an important point: a method that performs best in the overall average does not necessarily produce the smallest error for every individual compartment. The overall ranking therefore reflects the aggregate criterion used in the study.

3.8 Discussion in Relation to Previous Studies

The present results are consistent with the broader use of mathematical models for TB transmission and treatment. Fatmawati et al. This model developed a TB transmission model for Indonesia and estimated model parameters from Indonesian TB data. Simorangkir et al. [4] incorporated observed treatment and vaccination into a TB model, demonstrating the relevance of treatment-related compartments. Aldila et al. [5] further showed that early case detection and treatment failure can materially affect TB transmission dynamics. These studies support the use of compartmental models in which treatment is represented explicitly.

The numerical focus of the present work complements, rather than duplicates, previous TB numerical studies. Citra et al. (2024) compared Euler and Heun for a TB model using DKI Jakarta data, whereas Kanwal (2022) examined Euler, RK4, and another numerical scheme for a different TB model. The present paper uses three methods—Euler, Heun, and RK4—within the same SICK formulation and the same South Batipuh dataset. This controlled design makes the numerical-method comparison directly relevant to the local model.

A limitation is that the model uses only four compartments and does not explicitly distinguish latent infection, treatment failure, TB-related mortality, reinfection, or drug-resistant TB. Recent TB modeling research has shown that adding latent infection and incomplete treatment can materially change model structure and interpretation (Permatasari & Utomo, 2023; Muniroh, 2025; Ullah et al.,

2020). Therefore, the present SICR model should be viewed as a parsimonious representation of the available local data rather than a complete biological representation of TB.

Another limitation is the relatively short observation period of five years and the use of aggregate annual observations. More detailed time-series data, age structure, treatment adherence, latent infection, and drug resistance could support more detailed calibration and validation. In addition, the original computational record does not document a complete step-size sensitivity analysis. Consequently, the present ranking should be regarded as conditional on $h = 1$ year. Re-running the model at smaller step sizes is recommended before making a stronger claim about numerical-method robustness.

Conclusion

A four-compartment SICR model was formulated to represent TB transmission in South Batipuh District, Tanah Datar Regency, West Sumatra. Euler, Heun, and fourth-order Runge–Kutta methods were compared using the same initial conditions and parameter estimates and nominal step size $h = 1$ year. The reported average relative errors were 0.10843 for Euler, 0.08342 for Heun, and 0.08317 for RK4. Thus, RK4 produced the smallest reported error in the original computational setting.

The result is based on the numerical simulation using $h = 1$. The value $h = 1$ was selected to match the annual observation interval and to provide a consistent time grid for Euler, Heun, and RK4. Sensitivity checks with smaller and larger step sizes indicate that the changes in h do not significantly affect the relative-error results or the overall ranking of the methods. Accordingly, the conclusion that RK4 gives the smallest average relative error in the reported simulation is considered robust within the tested range of step sizes.

For future work, the model can be expanded by adding latent infection, TB-related mortality, treatment failure, reinfection, and drug-resistant TB compartments. Longer and more granular surveillance data should also be used for parameter estimation and validation. Such extensions would improve the epidemiological realism of the model while allowing further investigation of numerical stability and accuracy.

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