Journal of Applied Mathematics and Modelling
https://ejournal.cibnusantara.org/jamm
E-ISSN: 3090-5524

Transient Response Analysis of a Series RLC Circuit Using the Laplace Transform and MATLAB

Muhammad Wanda, Rolan Abdiansyah Depri, Talita Dwi Yanti, Ragil Pernanda, and Yusuf Saputra*

Electrical Engineering, Padang State Polytechnic, West Sumatera, Indonesia

* Correspondence e-mail: yusufsaputra@gmail.com

Article history:
Received : January 5th, 2026
Accepted : March 10th, 2026
Published : May 5th, 2026

DOI: https://doi.org/10.64570/

Citation: To be added by editorial staff during production.

Abstract: Transient response analysis is essential in electrical engineering for understanding the dynamic behavior of electrical circuits during switching and energy-transfer processes. This study aims to analyze the transient response of a series resistor–inductor–capacitor (RLC) circuit using the Laplace Transform and to numerically verify the analytical results using MATLAB. A quantitative analytical approach combined with numerical simulation was employed. The governing differential equation was formulated using Kirchhoff's Voltage Law and transformed into the Laplace domain to derive the circuit transfer function. The circuit was evaluated under a 12-V step input with an inductance of 0.5 H and a capacitance of 500 μF, while the resistance was varied at 10, 20, 40, 63.25, and 80 Ω to represent different damping conditions. The transient-response characteristics were evaluated in terms of damping ratio, rise time, peak time, maximum overshoot, settling time, and steady-state voltage. The results showed that increasing resistance substantially reduced oscillation and maximum overshoot. At 10 Ω, the circuit exhibited an underdamped response with a maximum overshoot of 60.47%, whereas at the critical resistance of approximately 63.25 Ω, the overshoot was eliminated and the settling time decreased to 0.0922 s. Increasing the resistance further to 80 Ω resulted in an overdamped response with a longer settling time of 0.1350 s. The analytical and MATLAB results showed differences of less than 0.01% for the principal transient-response parameters. These findings demonstrate that the Laplace Transform provides an effective mathematical framework for characterizing series RLC transient behavior, while MATLAB provides reliable numerical verification and facilitates quantitative evaluation of transient-response characteristics.

Keywords: Laplace Transform; series RLC circuit; transient response; transfer function; MATLAB.

1. Introduction

Transient analysis plays an important role in electrical engineering because it describes the dynamic behavior of electrical systems immediately after switching operations or external disturbances. Unlike steady-state analysis, transient analysis focuses on the short-term response before the system reaches equilibrium. Excessive transient currents, voltage overshoots, and oscillations may reduce system reliability, accelerate component degradation, and affect the performance of electrical and electronic equipment. Therefore, transient analysis has become an essential aspect in the design of power electronic converters, electrical protection systems, communication circuits, and automatic control systems (Ogata, 2020; Dorf & Bishop, 2021; Okafor & Oyetoro, 2025).

Among various electrical circuit configurations, the resistor–inductor–capacitor (RLC) circuit represents one of the most fundamental second-order dynamic systems because it exhibits three characteristic responses, namely underdamped, critically damped, and overdamped. The transient behavior of an RLC circuit depends on the interaction between the energy stored in the inductor and capacitor and the energy dissipated by the resistor. Consequently, RLC circuits are widely used as mathematical models for studying dynamic systems, transfer functions, stability, and transient characteristics in electrical engineering education and industrial applications (Haška et al., 2021; Torriente-García et al., 2024).

The Laplace Transform is one of the most widely used mathematical techniques for solving linear differential equations in engineering. By transforming differential equations from the time domain into algebraic equations in the complex-frequency domain, the Laplace Transform simplifies the derivation of transfer functions and transient responses while preserving the dynamic characteristics of the system. Furthermore, modern computational software such as MATLAB enables analytical solutions to be verified through numerical simulation, making mathematical modeling more efficient and easier to interpret (MathWorks, 2026a; MathWorks, 2026b).

Several previous studies have investigated transient responses of RLC circuits using different analytical and computational approaches. Alizadeh et al. (2020) analyzed the transient behavior of fractional RLC circuits using the Caputo–Fabrizio derivative and demonstrated the effectiveness of analytical modeling for describing dynamic responses. Haška et al. (2021) investigated transient and steady-state responses of fractional-order RLC circuits and reported that circuit parameters significantly influenced damping characteristics. Luz and Rossini (2023) developed a mathematical model of a parallel RLC circuit based on Kirchhoff's laws and validated the analytical solution using MATLAB and LTspice. Likewise, Zorica and Cvetićanin (2023) proposed a generalized transient model for dissipative and generative RLC circuits and highlighted the importance of mathematical analysis in predicting system behavior. More recently, Lesnussa et al. (2025) demonstrated that Laplace Transform solutions for RL, RC, and series RLC circuits were consistent with MATLAB simulations, while Okafor and Oyetoro (2025) extended transient analysis by incorporating Heaviside step functions to investigate several damping conditions.

Although these studies have confirmed the effectiveness of the Laplace Transform and numerical simulation for transient analysis, most of them emphasize either mathematical derivation or computational implementation separately. Only a limited number of studies quantitatively compare analytical solutions and MATLAB simulations using transient-performance indices such as rise time, peak time, maximum overshoot, settling time, and steady-state response. Moreover, the influence of resistance variation on the transient characteristics of a conventional series RLC circuit has not been comprehensively discussed using an integrated analytical–simulation framework.

Therefore, this study proposes an integrated approach for analyzing the transient response of a series RLC circuit using the Laplace Transform and validating the analytical model through MATLAB simulation. Unlike previous studies, this work combines complete mathematical derivation with numerical validation while investigating the influence of resistance on damping characteristics and transient-response performance. The findings are expected to provide a simple yet systematic methodology for analyzing second-order electrical systems and to support the teaching of applied mathematics and electrical circuit analysis in undergraduate electrical engineering programs.

2. Methods

This study employed a quantitative analytical approach combined with numerical simulation to investigate the transient response of a series resistor–inductor–capacitor (RLC) circuit. The analytical model was developed using Kirchhoff's Voltage Law and solved using the Laplace Transform, while MATLABwas employed to verify the mathematical response. The use of Laplace-domain modeling is appropriate because it transforms linear differential equations into algebraic expressions and facilitates the derivation of transfer functions under zero initial conditions (MathWorks, 2026a, 2026b). Similar analytical approaches have also been employed to investigate transient responses in classical and generalized RLC circuits (Alizadeh et al., 2020; Haška et al., 2021; Zorica & Cvetićanin, 2023).

Mathematical Model

The system analyzed in this study was a series RLC circuit consisting of a resistor R, an inductor L, and a capacitor C, connected to a direct-current step-voltage source. The capacitor voltage $v_C(t)$ was selected as the output variable. The circuit model was formulated using Kirchhoff's Voltage Law, which has been widely applied to construct governing differential equations for transient electrical-circuit analysis (Alizadeh et al., 2020; Haška et al., 2021).

According to Kirchhoff's Voltage Law,

$$v_s(t) = v_R(t) + v_L(t) + v_C(t), \quad (1)$$

The voltage across the resistor is

$$v_R(t) = Ri(t). \quad (2)$$

The voltage across the inductor is

$$v_L(t) = L\frac{di(t)}{dt}. \quad (3)$$

The relationship between the current and capacitor voltage is

$$i(t) = C\frac{dv_C(t)}{dt}. \quad (4)$$

Substituting Equations (2)–(4) into Equation (1) produces the governing differential equation

$$LC\,\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = v_s(t). \quad (5)$$

The circuit was analyzed under zero initial conditions

$$v_C(0) = 0,\ i(0) = 0. \quad (6)$$

Laplace Transform Analysis

The Laplace Transform was applied to convert the differential equation in the time domain into an algebraic equation in the complex-frequency domain. This method permits the initial conditions and input function to be incorporated directly into the mathematical solution (MathWorks, 2026a). Laplace-transform methods have also been used in previous studies to determine transient responses and impulse responses of RLC systems (Alizadeh et al., 2020; Zorica & Cvetićanin, 2023).

Applying the Laplace Transform to Equation (5) under zero initial conditions gives

$$LCs^2V_C(s) + RCsV_C(s) + V_C(s) = V_s(s)\ . \tag{7}$$

The transfer function between the capacitor voltage and the source voltage is therefore

$$G(s) = \frac{V_C(s)}{V_s(s)} = \frac{1}{LCs^2+RCs+1}. \tag{8}$$

A transfer function represents the input–output relationship of a linear time-invariant system in the Laplace domain and is commonly derived under zero initial conditions (MathWorks, 2026b). The dynamic characteristics of the circuit were identified using the natural frequency

$$\omega_n = \frac{1}{\sqrt{LC}}, \tag{9}$$

and the damping ratio

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}\ . \tag{10}$$

The system response was classified as underdamped, critically damped, or overdamped according to the value of ζ. This classification was used only to organize the simulation scenarios, the numerical values and their interpretations are presented in the Results and Discussion section.

Circuit Parameters and Simulation Procedure

The analytical model was implemented in MATLAB. MATLAB was used to construct the transfer function and calculate the response indices, whereas Matlab was used to represent the circuit model through interconnected blocks. The Transfer Function block is suitable for simulating continuous linear systems defined by numerator and denominator polynomial coefficients, provided that zero initial conditions are assumed (MathWorks, 2026c). The circuit parameters used in the analysis are listed in Table 1.

Table 1. Series RLC circuit parameters

Parameter	Symbol	Value
Step-input voltage	V_0	12 V
Inductance	L	0.5 H
Capacitance	C	500 μF
Initial capacitor voltage	$v_C(0)$	0 V
Initial inductor current	$i(0)$	0 A
Simulation duration	t	1 s

The Matlab model consisted of a step block, a transfer function block, and a scope block. The step block generated the input voltage, the transfer function block represented Equation (8), and the scope block displayed the capacitor-voltage response. Similar integration of theoretical circuit models and computational or experimental representations has been used to validate RLC-circuit behavior in engineering and physics education (Torriente-García et al., 2024).

Resistance was treated as the varied parameter, while the inductance, capacitance, input voltage, initial conditions, and simulation duration were maintained constant. The selected resistance values are shown in Table 2.

Table 2. Resistance variation used in the simulation

Scenario	Resistance (R)
1	$10\ \Omega$
2	$20\ \Omega$
3	$40\ \Omega$
4	$63.25\ \Omega$
5	$80\ \Omega$

The simulation procedure consisted of the following operational stages. First, the values of R, L, and C were entered into the transfer-function model. Second, a 12-V step input was applied to the system. Third, the capacitor-voltage response was simulated for one second. Fourth, the simulation was repeated for each resistance value. Finally, the response data were exported from MATLAB for comparison with the analytical results.

Data Analysis

The analytical and simulation responses were evaluated using rise time, peak time, maximum overshoot, settling time, and steady-state value. These indices were extracted from the simulated response using MATLAB step-response analysis.

The analytical and simulation values were compared using the percentage error

$$E = \left|\frac{X_{sim} - X_{analytical}}{X_{analytical}}\right| x\ 100\%\ , \tag{11}$$

where $X_{analytical}$ denotes the value obtained from the Laplace-based model and X_{sim} denotes the corresponding value obtained from MATLAB.

The data analysis was conducted by deriving the circuit differential equation, obtaining the transfer function using the Laplace Transform, implementing the transfer function in MATLAB, simulating the response for each resistance value, extracting the transient-response indices, and calculating the percentage error between the analytical and simulation results. The resulting numerical values, response graphs, and interpretations were reserved for the Results and Discussion section.

3. Results and Discussion

Mathematical Characteristics of the Series RLC Circuit

The transient response of the series RLC circuit was analyzed using the parameters $\boldsymbol{L} = 0.5\ H, C = 500\ \mu F$, and $V_0 = 12\ V$. Resistance was varied from 10 Ω to 80 Ω to represent different damping conditions.

The natural frequency of the circuit was calculated as

$$\omega_n = \frac{1}{\sqrt{(0.5)(500\times10^{-6})}} = 63.2456 \text{ rad/s} .$$

The critical resistance was determined using

$$R_c = 2\sqrt{\frac{0,5}{500\times10^{-6}}} = 63.2456\ \Omega\ .$$

The value above indicates that resistance values below 63.2456 Ω produce an underdamped response, a resistance equal to 63.2456 Ω produces a critically damped response, and values above 63.2456 **Ω** produce an overdamped response. The damping ratios obtained for the five resistance scenarios are presented in Table 3.

Table 3. Dynamic characteristics of the series RLC circuit

R (Ω)	ζ	System Poles	Response Condition
10	0,1581	-10 $\pm$62.45 j	Underdamped
20	0,3162	-20 $\pm$60 j	Underdamped
40	0,6325	-40 $\pm$48.99 j	Underdamped
63,25	1	-63.25 , -63.25	Critically damped
80	1,2649	-128.99 , -31.01	Overdamped

All poles had negative real parts, indicating that every investigated configuration was stable. However, the nature of the poles changed with resistance. Complex-conjugate poles occurred at $R = 10, 20,$ and 40Ω, explaining the oscillatory responses under these conditions. At the critical resistance, the system had two equal real poles, whereas the 80 Ω circuit had two different negative real poles.

Transient Response for the Nominal Circuit

For the nominal resistance of 20Ω, the transfer function was obtained from

$$G(s) = \frac{4000}{s^2+40s+4000}\ .$$

For a 12-V step input,

$$V_s(s) = \frac{12}{s}\ ,$$

and the capacitor voltage in the Laplace domain is

$$V_C(s) = \frac{48000}{s(s^2+40s+4000)}\ .$$

The damping ratio and damped natural frequency for this circuit were $\zeta = 0.3162$ and

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 60 \text{ rad/s}.$$

The inverse Laplace Transform produced the capacitor-voltage response

$$v_C(t) = 12\left[1 - e^{-20t}\left(\cos(60t) + \frac{1}{3}\sin(60t)\right)\right]\ .$$

the capacitor-voltage response above shows that the response contains an exponentially decreasing oscillatory component. The term e^{-20t} controls the decay of the oscillation, while the trigonometric terms describe its oscillatory behavior. As time increases, the exponential term approaches zero and the capacitor voltage approaches the input voltage of 12V.

The theoretical peak time was 0.05236 s and The maximum percentage overshoot was 35.09 %. Consequently, the theoretical peak voltage was 16.21 V.

This result indicates that although the source voltage was only 12 V, the capacitor voltage temporarily reached approximately 16.21 V. This transient voltage increase results from the continuous exchange of stored energy between the inductor and capacitor.

Effect of Resistance on the Transient Response

The numerical step-response results for all resistance values are presented in Table 4. Rise time was evaluated between 10% and 90% of the steady-state value, while settling time was determined using the 2% tolerance criterion. These definitions are consistent with the default response-characteristic criteria implemented by MATLAB's function.

Table 4. Transient-response characteristics for different resistance values

$R(\Omega)$	Rise Time (s)	Peak Time (s)	Peak Voltage (s)	Overshoot (%)	Setting Time (s)
10	0.0183	0.0503	19.256	60.47	0.3659
20	0.0212	0.0524	16.211	35.09	0.1768
40	0.0306	0.0641	12.923	7.69	0.0948
63.25	0.0531	-	12	0	0.0922
80	0.0743	-	12	0	0.1350

These findings confirm that reducing overshoot does not necessarily guarantee the fastest overall response. Increasing resistance from 10 Ω to the critical value improved both damping and settling performance. However, increasing resistance beyond the critical value caused the rise and settling times to increase. Thus, the relationship between resistance and response speed is nonlinear.

Comparison of Analytical and Numerical Results

The analytical and numerical results for the nominal resistance are compared in Table 5.

Table 5. Comparison of analytical and numerical results for $R = 20\Omega$

Response parameter	Analytical result	Numerical result	Percentage difference
Damping ratio	0.3162	0.3162	0.00%
Natural frequency	63.2456 rad/s	63.2456 rad/s	0.00%
Damped frequency	60.0000 rad/s	60.0000 rad/s	0.00%
Peak time	0.05236 s	0.05236 s	(<0.01%)
Maximum overshoot	35.092%	35.092%	(<0.01%)
Peak voltage	16.211 V	16.211 V	(<0.01%)
Steady-state voltage	12.000 V	12.000 V	0.00%

The nearly identical results occurred because both calculations were based on the same differential equation, parameter values, input, and zero initial conditions. The analytical method generated an exact continuous-time expression, whereas the numerical procedure evaluated the response at discrete time points. Any small difference was therefore attributable to numerical precision, solver configuration, and time-step selection rather than to disagreement between the two models.

The agreement confirms that the Laplace Transform accurately represents the transient response of the series RLC circuit. It also demonstrates that MATLAB can be used as a computational validation instrument rather than as a replacement for mathematical modeling. The analytical expression explains the physical and mathematical structure of the response, while the numerical simulation facilitates visualization and extraction of response indices.

4. Conclusion

This study demonstrated that the Laplace Transform can accurately describe the transient response of a series RLC circuit and provide a clear mathematical representation of its dynamic behavior. The results showed that resistance strongly influenced the damping ratio, overshoot, rise time, and settling time of the system. Low resistance values produced underdamped responses with high oscillation and overshoot, while increasing resistance reduced the oscillation and improved the settling performance. A resistance value close to the critical resistance generated the most balanced response because the circuit reached the steady-state condition rapidly without overshoot, whereas an excessively large resistance produced a slower overdamped response. The close agreement between the analytical calculations and MATLAB simulations confirmed the validity of the developed mathematical model. Therefore, the integration of Laplace Transform analysis and numerical simulation provides an effective approach for investigating second-order electrical systems and can be applied as a practical learning method in applied mathematics and electrical engineering courses.

References

Alizadeh, S., Baleanu, D., & Rezapour, S. (2020). Analyzing transient response of the parallel RCL circuit by using the Caputo–Fabrizio fractional derivative. *Advances in Difference Equations, 2020*, Article 55. https://doi.org/10.1186/s13662-020-2527-0

Dorf, R. C., & Bishop, R. H. (2022). *Modern Control Systems* (14th ed.). Pearson.

Haška, K., Zorica, D., & Cvetićanin, S. M. (2021). Fractional RLC circuit in transient and steady-state regimes. *Communications in Nonlinear Science and Numerical Simulation, 96*, 105670. https://doi.org/10.1016/j.cnsns.2020.105670

Lesnussa, Y. A., Siahaya, J., & Waas, T. (2025). *Laplace transform analysis of RL, RC, and series RLC circuits using MATLAB validation*. *Paulus Computing Journal*, 4(1), 1–10.

Luz, A. P., & Rossini, L. A. (2023). *Mathematical modeling and analysis of a parallel RLC circuit via Laplace transform using MATLAB and LTspice*. *Proceedings of the Brazilian Conference on Engineering Education*, 45–53.

MathWorks. (2026a). *Solve differential equations using Laplace transform*. https://www.mathworks.com/help/symbolic/solve-differential-equations-using-laplace-transform.html

Ogata, K. (2010). *Modern Control Engineering* (5th ed.). Pearson.

Okafor, C. O., & Oyetoro, O. S. (2025). Transient and frequency response analysis of modified RLC circuits using Heaviside step functions. *FUDMA Journal of Sciences, 9*(2), 113–124.

Sharma, A., Gupta, R., & Verma, S. (2022). MATLAB-based transient response analysis of series RLC circuits under different damping conditions. *International Journal of Electrical and Computer Engineering, 12*(4), 3567–3575.

Torriente-García, I., Martí, A. C., Monteiro, M., Stari, C., Castro-Palacio, J. C., & Monsoriu, J. A. (2024). RLC series circuit made simple and portable with smartphones. *Physics Education, 59*(1), 015016. https://doi.org/10.1088/1361-6552/ad04fb

Zorica, D., & Cvetićanin, S. M. (2023). Dissipative and generative fractional RLC circuits in the transient regime. *Applied Mathematics and Computation, 459*, 128227. https://doi.org/10.1016/j.amc.2023.128227