Journal of Applied Mathematics and Modelling
https://ejournal.cibnusantara.org/jamm
E-ISSN: 3090-5524

Graph Coloring Application in Course Scheduling Using the Welch-Powell Algorithm (Case Study: Mathemat nanics Study Program, Dharma Andalas University)

Salsabilla Mirza[1*], Iswan Rina[1], Radhiatul Husna[2]

[1]Mathematics Department, Andalas Dharma University, West Sumatera Province, Indonesa
[2]Mathematics Department, Andalas University, West Sumatera Province, Indonesa

* Correspondence e-mail: salsabillamirza865@gmail.com

Article history:
Received : February 15th, 2026
Accepted : April 22th, 2026
Published : May 5th, 2026

DOI: https://doi.org/10.64570/jamm.v2i1.58

Citation: To be added by editorial staff during production.

Abstract: Course scheduling is a crucial and routine academic activity carried out by universities prior to the commencement of a new academic year, requiring precise arrangements to avoid timing conflicts for both lecturers and students. The course scheduling for the odd semester of the 2024/2025 academic year at the Mathematics Study Program of Dharma Andalas University (UNIDHA) faces high complexity in manually allocating various combinations of lecturers, classes, and courses. This study aims to optimize the scheduling system by utilizing the concept of graph coloring, specifically employing the Welch-Powell Algorithm. The research method involves transforming the lecturer's teaching assignment data into an undirected graph model. Course variations are modeled as 21 vertices, while potential scheduling conflicts due to identical lecturers or student groups are represented as edges. The Welch-Powell Algorithm is then applied by sorting the vertices based on the largest degree ordering to color them progressively. The results indicate that this graph representation successfully yields a chromatic number of $x(G) = 6$. Consequently, from a total of 9 lecturers teaching 21 courses, the entire academic schedule is optimally distributed into 6 distinct color groups without any conflicts. These color groups are successfully implemented into 6 academic working days (Monday to Saturday) with precise time-slot allocations corresponding to the respective credit semester units (SKS). The study concludes that the application of graph coloring using the Welch-Powell Algorithm is highly effective, objective, and systematic in resolving course scheduling conflicts within the Mathematics Study Program at UNIDHA.

Keywords: Graph, Graph Coloring, Vertex, Course Scheduling, Welch-Powell Algorithm, Chromatic Number.

1. Introduction

Lectures are a key academic activity in higher education that require structured planning, one of which is through the preparation of a course schedule at the beginning of the semester (Welch & Powell, 1967). However, the manual schedule preparation process often faces high complexity due to the increasing number of courses, space constraints, and the potential for conflicting schedules of lecturers teaching more than one class (West, 2001). This allocation optimization problem can be mathematically transformed into graph theory, specifically the problem of vertex coloring (West, 2001). The main problem identified in a case study of the Undergraduate Mathematics Study Program at Dharma Andalas University (UNIDHA) in the odd semester of the 2024/2025 academic year is the congestion of schedules and time constraints due to several courses being taught by the same lecturer or being taught to the same class group. If completed conventionally, this process is time-consuming and prone to errors (trial and error). Therefore, a systematic approach based on mathematical algorithms is needed to model lecture components as vertices and potential schedule conflicts as edges, so that clashes can be minimized objectively.

Several previous studies have attempted to solve scheduling optimization problems using graph theory. Research based on graph coloring algorithms has generally been grounded by Welch and Powell (1967) in solving timetabling problems efficiently. In the context of modern educational institutions, several similar studies have also utilized adjacency matrix representations to map schedule conflicts (Munir, 2010). However, there is a research gap where most previous studies applied these algorithms at the macro university scale, assuming classroom availability as the primary constraint. Meanwhile, at a more specific study program scale, such as the Mathematics Study Program at UNIDHA, the characteristics of the problem found lie in the high frequency of tandem courses (taught by the same combination of lecturers) and the distribution of credits that vary between 2 and 3 credits. The novelty of the current research lies in modeling these local constraints into a descending degree order (Largest Degree Ordering) that is realistically adjusted to the institution's work schedule distribution. The urgency or significance of solving this problem is to reduce schedule preparation time, optimize the operational performance of the study program, and ensure the smooth running of the teaching and learning process without interruption from schedule conflicts. Based on the background of the problem, the purpose of this study is to find out how to compile, model, and color graphs to produce a draft of the odd semester lecture schedule for the 2024/2025 academic year in the UNIDHA Mathematics Study Program that is free from conflicts using the Welch-Powell Algorithm.

To achieve the research objectives, this analysis is guided by a discrete mathematics framework that focuses on graph structure and combinatorial optimization (Munir, 2010). The main theories underlying all these experiments include the definition of a formal graph as a pair of vertex and edge sets, as well as the principle of graph coloring that aims to assign different colors to neighboring vertices using a minimum number of colors (West, 2001). In addition, the Welch-Powell Algorithm is used as the main deterministic instrument to sort the vertex degrees from highest to lowest to obtain the chromatic number ($x(G)$) that represents the minimum number of time slots or days required (Welch & Powell, 1967; West, 2001). In more detail, the supporting theories underlying this modeling will be explained in the following sub-headings.

Graph

A graph G is formally defined as a pair of sets (V, E), written as $G = (V, E)$. In this representation, V is a finite and non-empty set of vertices, written as $V = (v_1, v_2, v_3, ..., v_n)$, while E is the set of edges connecting a pair of vertices, written as $E = (e_1, e_2, e_3, ..., e_m)$ (West, 2001). Two vertices in a graph that are directly connected by an edge are called adjacent (Munir, 2010). The number of edges adjacent to a vertex determines the degree of that vertex, denoted as $d(v)$ (Munir, 2010). The characteristics of the adjacency and degree relationships are the basic foundation for mapping the presence or absence of conflicts between components in a scheduling system.

Graph Coloring

Graph coloring is one of the important optimization problems in graph theory which is generally divided into three types, namely vertex coloring, edge coloring, and region coloring (West, 2001). The main focus in scheduling problems is vertex coloring, which is to assign a color to each vertex in such a way that no two neighboring vertices have the same color (West, 2001). The ultimate goal of this problem is not just coloring, but minimizing the number of different colors used, which is known as the chromatic number and is denoted by $x(G)$ (West, 2001). Through the chromatic number, determining the minimum number of time slots that is efficient and conflict-free can be achieved (West, 2001).

Welch-Powell Algorithm

The Welch-Powell algorithm is a heuristic method used to color the vertices of a graph precisely, simply, and easily (West, 2001). This algorithm works based on the Largest Degree Ordering (LDO) principle, where the coloring process begins by sorting all vertices by their degree in descending order from highest to lowest (West, 2001). The implementation steps include: (1) sorting the vertices in descending order; (2) assigning the first color to the vertex with the highest degree and subsequent vertices that are not adjacent to it; (3) repeating the process with a new color for the next highest-degree vertex that has not yet been colored; and (4) iterating the addition of colors until all vertices in the graph have been colored (West, 2001).

2. Methods

This study applies a quantitative-objective approach using formal structural modeling within discrete mathematics, specifically leveraging graph theory and combinatorial optimization. The research context is centered on the academic administration of the Mathematics Study Program at Dharma Andalas University (UNIDHA), Padang. The subjects of this research comprise the scheduling configuration for the odd semester of the 2024/2025 academic year, involving a dataset of 9 lecturers and 21 unique courses. The primary data source is the official administrative lecturer teaching assignment document issued by the department.

To ensure the quantitative trustworthiness criteria established by Given and Saumure (2008), this study satisfies four aspects: internal validity by mapping actual courses directly as vertices (V) and schedule conflicts as edges (E); objectivity by using fixed institutional data completely free from personal bias; reliability through a deterministic algorithmic process that yields identical results ($x(G) = 6$) under replication; and generalizability by providing an optimization framework that can be adapted by other academic programs facing identical multi-variable scheduling barriers.

The data analysis framework progresses through a systematic procedure directly extracted from the original flowchart of the research. The operational stages of this study are visually mapped in Figure 1 below:

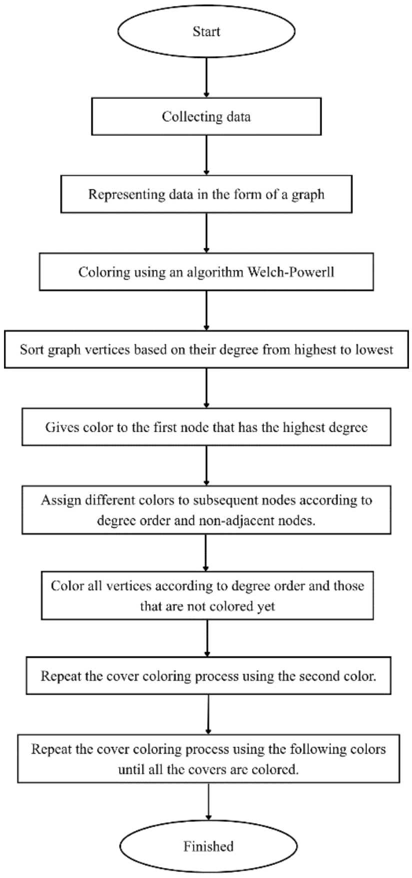

Figure 1. Research Flowchart

Referring to the data analysis sequence shown in Figure 1, the operational steps are concisely elaborated as follows:

1. Data Collection and Assumptions: Primary administrative data was collected regarding the teaching tasks for the odd semester, covering four active student cohorts (2021, 2022, 2023, and 2024). Two underlying baseline assumptions were applied: the scheduling is strictly bounded to this single semester, and classroom spaces are assumed to be constantly available, thereby isolating the operational constraints purely to timeline and lecturer allocations.
2. Graph Model Representation: The parameters were formulated into an undirected graph $G = (V, E)$. The fields of teaching assignments (the courses) were represented as a set of 21 vertices (V). The timeline collisions—occurring when a lecturer teaches multiple classes or student cohorts overlap—were mathematically mapped as connecting edges (E).
3. Welch-Powell Graph Coloring: The computational optimization was executed using the Welch-Powell algorithm through four precise sub-steps:
    a. All 21 vertices were sorted in descending order based on their degree values $d(v)$ using the Largest Degree Ordering (LDO) method.
    b. The primary color was assigned to the first vertex with the highest degree value.
    c. The same color was sequentially applied downward to subsequent uncolored vertices that are not adjacent (do not share an edge) with the first vertex.
    d. The algorithm reset its pointer to the next highest available uncolored vertex, introducing

a new distinct color, and repeated the loop until all vertices were successfully labeled

4. Finish and Schedule Mapping: The process terminated once all 21 vertices were fully colored, defining a final chromatic number $x(G) = 6$. These 6 independent color boundaries were directly mapped into the 6 active operational working days of the university (Monday to Saturday), establishing a perfectly distributed, conflict-free academic timetable.

3. Results and Discussion

Representing Data in Graph Form

The data used are teaching decrees for lecturers in the Mathematics Study Program at Dharma Andalas University during the odd semester of the 2024/2025 academic year. The data includes a list of lecturers' names, courses, and teaching hours. The variety of courses and lecturers is expressed in vertex code, modeled in a graph and represented as nodes, which represent constraints to be met. The constraints are requirements to ensure that the course schedules used in the odd semester of the 2024/2025 academic year do not conflict. Lecturers who have the same number of teaching hours as other lecturers represent the edges connecting the nodes.

Definition of each vertex

1. Courses taught by two lecturers are considered tandem courses. The number of credits for these courses is multiplied by 2.
2. Tandem courses are taught by only one lecturer.
3. The number of credits available at Dharma Andalas University is only 2-3 credits.

The vertex code, adjacency, and coloring order can be presented in the table below.

Table 1. Vertex Code, Adjacency, and Coloring Order

Vertex Code	Adjacency	Degrees	Coloring Order
V1	V2, V5, V6, V14, V15	5	1
V2	V1, V5, V6, V14, V15	5	2
V3	V4	1	12
V4	V3	1	13
V5	V1, V2, V6, V14, V15	5	3
V6	V1, V2, V5, V14, V15	5	4
V7	V8,V9,V13,V21	4	7
V8	V7,V9,V13,V21	4	8
V9	V7, V8, V13, V21	4	9
V10	V11	1	14
V11	V10	1	15

V12	V20	1	16
V13	V7, V8, V9, V21	4	10
V14	V1, V2, V5, V6, V15	5	5
V15	V1, V2, V5, V6, V14	5	6
V16	-	0	20
V17	V18	1	17
V18	V17	1	18
V19	-	0	21
V20	V12	12	19
V21	V7, V8, V9, V13	4	11

In the table above, we can see that there are neighboring nodes and there are also non-neighboring nodes. The degree of these nodes is determined by the number of their neighbors, so we can determine which nodes have the highest to lowest degree and determine the coloring order of these nodes and the data representation obtained is modeled in the form of a graph as follows:

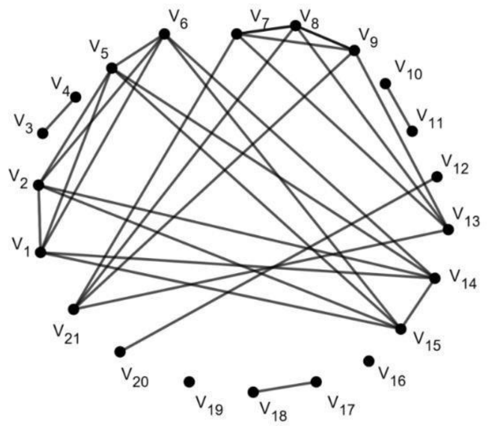

Figure 2. Uncolored graph

From the figure above, we can see that some vertices have edges connecting them, while others do not. Therefore, based on Table 1. and Figure 2, we can explain that:

1. Vertex V1 has 5 neighbors: V2, V5, V6, V14, and V15.
2. Vertex V2 has 5 neighbors: V1, V5, V6, V14, and V15.
3. Vertex V3 has 1 neighbor: V4.
4. Vertex V4 has 1 neighbor: V3.
5. Node V5 has 5 neighbors: V1, V2, V6, V14, and V15.
6. Node V6 has 5 neighbors: V1, V2, V5, V14, and V15.
7. Node V7 has 4 neighbors: V8, V9, V13, and V21.
8. Node V8 has 4 neighbors: V7, V9, V13, and V21.

9. Node V9 has 4 neighbors: V7, V8, V13, and V21.
10. Node V10 has 1 neighbor: V11.
11. Node V11 has 1 neighbor: V10.
12. Node V12 has 1 neighbor: V20.
13. Node V13 has 4 neighbors: V7, V8, V9, and V21.
14. Vertex V14 has 5 neighbors: V1, V2, V5, V6, and V15.
15. Vertex V15 has 5 neighbors: V1, V2, V5, V6, and V14.
16. Vertex V16 has no neighbors because there are no edges connecting them.
17. Vertex V17 has 1 neighbor: V18.
18. Vertex V18 has 1 neighbor: V17.
19. Vertex V19 has no neighbors because there are no edges connecting them.
20. Vertex V20 has 1 neighbor: V12.
21. Vertex V21 has 4 neighbors: V7, V8, V9, and V13. The number of neighbors of each vertex is the degree used to sort the coloring from highest to lowest.

Aplication of Welch- Powell Algorithm

The course scheduling in the Mathematics study program at Dharma Andalas University for the odd semester of 2024/2025 uses the Welch-Powell algorithm for graph coloring, which begins by sorting vertices from highest to lowest degree. The process continues with gradual coloring of non-adjacent vertices, repeating until all vertices are colored. The nodes, their sorted degrees, and the colors of each node are presented in the table below.

Table 2. Nodes, sorted degrees, and colored

Nodes	Degrees	Color	Noted	Degrees	Color
V1	5	Purple	V3	1	Purple
V2	5	Pink	V4	1	Pink
V5	5	Green	V10	1	Green
V6	5	Blue	V11	1	Yellow
V14	5	Yellow	V12	1	Red
V15	5	Red	V17	1	Purple
V7	4	Purple	V18	1	Pink
V8	4	Pink	V20	1	Green
V9	4	Green	V16	1	Blue
V13	4	Blue	V19	1	Red
V21	4	Yellow			

Incremental Graph Coloring

Based on Table 2. it can be applied to the image in the graph model that has been colored according to the number of chromatic numbers, namely 6 and given 6 colors and the results of the graph that has been colored based on the chromatic number are obtained in the image below.

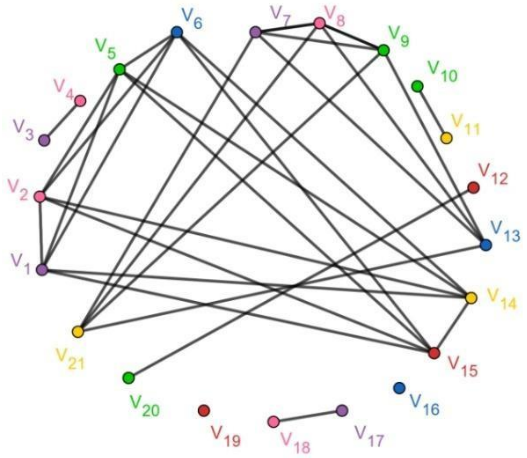

Figure 3. Colored graph

The minimum number of colors required, or chromatic number, is X(G) = 6. The graph coloring results, i.e., the chromatic number, of the 9 lecturers teaching 21 courses, can be scheduled within 6 days of teaching in the Mathematics Study Program at Dharma Andalas University.

Nodes with the same color indicate that the subjects can be scheduled in the same class, while edges that connect or connect the nodes indicate that the lecturers cannot teach in the same class. Therefore, a course can be scheduled in the same class on a single day and in different classes on different days, thus avoiding conflicts.

The following are the results of the subjects that can be scheduled in the same class:

1. Nodes V1, V3, V7, and V17 can be scheduled in the same class because they have the same node color, namely purple.
2. Nodes V2, V4, V8, and V18 can be scheduled in the same class because they have the same node color, namely pink.
3. Nodes V5, V9, V10, and V20 can be scheduled in the same class because they have the same node color, green.
4. Nodes V6, V13, and V16 can be scheduled in the same class because they have the same node color, blue.
5. Nodes V14, V11, and V21 can be scheduled in the same class because they have the same node color, yellow.
6. Nodes V12, V15, and V19 can be scheduled in the same class because they have the same node color, red.

Based on the above description, a course schedule can be prepared for the teaching and learning process in the odd semester of the 2024/2025 academic year in the Mathematics Study Program at Dharma Andalas University (UNIDHA). The division of lecture time is adjusted according to the provision that 1 credit unit is equivalent to 45 minutes, so that a 2-credit unit course is conducted for 90 minutes and a 3-credit unit course for 135 minutes. With this arrangement, all courses can be implemented systematically and without conflicting with each other.

4. Conclusion

Based on the results of data representation and course scheduling in the Mathematics Study Program at Dharma Andalas University, it can be concluded that the Welch-Powell algorithm can create a course schedule by representing data on the distribution of lecturers' tasks in the Mathematics Study Program at Dharma Andalas University during the teaching and learning process in the odd semester of the 2024/2025 academic year into a graph and generating chromatic numbers of six colors, which are represented as the days for the lecturers' teaching schedules in each class.

References

Bettinelli, A., Cacchiani, V., Roberti, R., & Toth, P. (2015). An overview of curriculum-based course timetabling. TOP, 23, 313-349. https://doi.org/10.1007/s11750-015-0366-z .

Bondy, J. A., & Murty, U. S. R. (2008). Graph Theory. London: Springer. Munir, R. (2010). Matematika diskrit. Bandung: Informatika.

Burke, E. K., Meisels, A., Petrovic, S., & Qu, R. (2007). A graph-based hyper-heuristic for timetabling problems. European Journal of Operational Research, 176(1), 177-192. https://doi.org/10.1016/j.ejor.2005.08.012 .

Chartrand, G., & Zhang, P. (2012). A First Course in Graph Theory. Dover.

Chen, M., Werner, F., & Shokouhifar, M. (2023). Mathematical modeling and exact optimizing of university course scheduling considering preferences of professors. Axioms, 12(5), 498. https://doi.org/10.3390/axioms12050498.

Cormen, T. H., et al. (2009). Introduction to Algorithms (3rd ed.). MIT Press. Harary, F. (1969). Graph Theory. Addison-Wesley.

Given, L. M., & Saumure, K. (2008). The SAGE Encyclopedia of Qualitative Research Methods. Sage.

Gross, J. L., & Yellen, J. (2006). Graph Theory and Its Applications (2nd ed.). Boca Raton: Chapman & Hall/CRC. Jensen, T. R., & Toft, B. (2011). Graph Coloring Problems. New York: Wiley.

Jensen, T. R., & Toft, B. (2011). Graph Coloring Problems. Wiley.

Klimošová, T., Malík, J., Masařík, T., Novotná, J., Paulusma, D., & Slívová, V. (2020). Colouring (Pr+Ps)-free graphs. Algorithmica, 82, 1833-1858. https://doi.org/10.1007/s00453-020-00675-w .

Lewis, R. M. R. (2021). Guide to graph colouring: Algorithms and applications (2nd ed.). Springer. https://doi.org/10.1007/978-3-030-81054-2 .

Munir, R. (2010). Matematika Diskrit. Informatika.

Tassopoulos, I. X., Iliopoulou, C. A., Katsaragakis, I. V., & Beligiannis, G. N. (2023). An effective local particle swarm optimization-based algorithm for solving the school timetabling problem. Algorithms, 16(6), 291. https://doi.org/10.3390/a16060291 .

Welch, T., & Powell, M. (1967). An upper bound for the chromatic number of a graph and its application to timetabling problems. The Computer Journal, 10(1), 85–86.

West, D. B. (2001). Introduction to Graph Theory (2nd ed.). New Jersey: Prentice Hall.