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Optimizing The Production Time of The Max-Plus Time Invariant (SLMI) Linier Algebraic System in The Production Of Naziza Potato Donuts in Padang City

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Abstract: In the era of globalization, business development is increasingly high, marked by the emergence of many MSMEs. With the emergence of so many MSMEs, the demands faced by MSME owners are getting higher, which has resulted in increasingly tighter and more competitive competition in the business world. One of them is MSMEs in the culinary sector, Naziza's Potato Donuts MSME is one of the MSMEs operating in the culinary sector, which is also facing this problem. Naziza's Potato Donut MSMEs must take various steps to survive, one of which is by improving service. The service that can be provided is fulfilling product orders on time and in the appropriate quantity. Based on this problem, Max-Plus Algebra is expected to be a way to optimize production time in the Naziza's Potato Donut production system, so that production time can be used effectively and efficiently. Based on the Max-Plus Algebra method, the optimal time for input (entering materials) and time for output (completion of production time) is obtained. So, from the Max-Plus Algebra calculations, the optimal time for MSME production of Naziza's Potato Donuts is obtained 4 times.

Keywords: Max-Plus Algebra Method, One Input One Output (SISO), Optimization

1. Introduction

In the era of globalization, the development of the business world is getting higher, marked by the emergence of many new MSMEs. MSMEs stand for Micro, Small and Medium Enterprises. Basically, MSMEs are a business or business carried out by individuals, groups, and small business entities, as well as households. MSMEs have long been the backbone of the economy in many countries, including Indonesia. MSMEs have an important role in creating jobs, driving economic growth, and improving people's welfare (Vinatra, 2023). Therefore, many MSMEs have emerged and developed in Indonesia, one of which is in the culinary/food sector. Naziza Potato Donut MSMEs are one of the MSMEs engaged in the culinary sector. These MSMEs have been running and developing for 3 years.

With the number of MSMEs, the demands faced by MSME owners are getting higher which causes competition to be fiercer and more competitive in the business world (Manambing et al., 2018). The same is the case faced by Naziza’s Potato Donut MSMEs. Naziza’s Potato Donut MSMEs must do various ways to survive, one of which is by improving services. The service that can be done is to fulfill product orders on time and in appropriate quantities. Thus, the hope of MSMEs is to increase the profits of these MSMEs.

Naziza's Potato Donut Production activities are related to the effectiveness of the use of time and the amount of labor. Based on these problems, Algebra Max-Plus is expected to be a way to optimize production time in the Naziza’s Potato Donut production system, so that production time can be used effectively and efficiently.

Related to this problem, the Max-Plus Algebra theory is one of the theoretical studies that can be used for modeling, analysis, and control in production systems. The main reason for using Algebra Max-Plus is because of its ease in completing the synchronization process. Max-Plus Algebra Synchronization has several advantages in optimizing production systems. In addition, Algebra Max-Plus has also been used well for modeling and analyzing algebraically.

Based on the description above, the author is interested in studying the optimization of the production system of Naziza’s Potato Donut MSMEs which uses the Max-Plus Linear Algebra System. In this study, data on the process of making potato donuts and the time was used. So the results of this research are in the form of an optimal schedule of the production process of Naziza's potato donuts.

2. Methods

A semiring $(\mathbf{S}, +, \times)$ is an empty set $\mathbf{S}$ equipped with two binary operations $+$ and $\times$, which fulfills the following axiom (Rudhito, M.A, 2020) :

i. $(\mathbf{S}, +)$ is a commutative semigroup with a neutral element $\mathbf{0}$, i.e. $\forall a, b, c \in \mathbf{S}$:

$$(a + b) + c = a + (b + c),$$

$$a + b = b + a,$$

$$a + \mathbf{0} = a. \tag{1}$$

ii. $(\mathbf{S}, \times)$ is a semigroup with a unit element **of 1**, i.e. $\forall \mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbf{S}$:

$$(a \times b) \times c = a \times (b \times c),$$

$$a \times \mathbf{1} = \mathbf{1} \times a = a \tag{2}$$

iii. The neutral element $\mathbf{0}$ is the absorbent element of the operation, namely $\times, \forall \mathbf{a} \in \mathbf{S}$:

$$a \times \mathbf{0} = \mathbf{0} \times a = \mathbf{0} \tag{3}$$

iv. Operation $\times$ operations on $+$, i.e. $\forall a, b, c \in \mathbf{S}$:

$$(a + b) \times c = (a \times c) + (b \times c),$$

$$a \times (b + c) = (a \times b) + (a \times c). \tag{4}$$

Given with $\mathbf{R}_\varepsilon \coloneqq \mathbf{R} \cup \{\boldsymbol{\varepsilon}\}$ with $\mathbf{R}$ is the set of all real numbers and $\varepsilon \coloneqq -\infty$. On $\mathbf{R}_\varepsilon$ is defined the following operation.

$$a \oplus b \coloneqq \max(a, b) \text{ and } a \otimes b \coloneqq a + b., \forall a, b \in \boldsymbol{R}_\varepsilon \tag{5}$$

Suppose $2 \oplus 1 \coloneqq \max(2,1)$ and $-3 \otimes 4 \coloneqq -3 + 4 = -1$.

$(\boldsymbol{R}_\varepsilon, \oplus, \otimes)$ is a semiring with a neutral element $\varepsilon = -\infty$ and a unit element $e = 0$, because for each $a, b, c \in \boldsymbol{R}_\varepsilon$ applies :

1. $a \oplus b = \max(a, b) = \max(b, a) = b \oplus a$,
2. $(a \oplus b) \oplus c = \max(\max(a, b), c) = \max(a, b, c) = \max(a, \max(b, c)) = a \oplus (b \oplus c)$,
3. $(a \otimes b) \otimes c = (a + b) + c = a + (b + c) = a \otimes (b \otimes c)$,
   $a \otimes e = a + 0 = a = 0 + a = e \otimes a$,
4. $a \otimes \varepsilon = a + (-\infty) = -\infty = (-\infty) + a = \varepsilon \otimes a$.
5. $(a \oplus b) \otimes c = \max(a, b) + c = \max(a + c, b + c) = (a \otimes b) \oplus (b \otimes c)$
   $a \otimes (b \oplus c) = a + \max(b, c) = (a + b, a + c) = (a \otimes b) \oplus (a \otimes c)$.

Algebra Max-Plus denoted by $\mathbb{R}_{\max}$ being defined as $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$, where $\mathbb{R}$ is a set of real numbers and is accompanied by two binary operations $\oplus$ i.e. the maximum operation and $\otimes$ is a regular sum operation (Carnia et al., 2023).

Invariant Time Max-Plus Linear System

The Max-Plus Time Invariant Linear System can be expressed by the following equation:

$$\begin{aligned} \mathbf{x}(k+1) &= A \otimes \mathbf{x}(k) \oplus B \otimes \mathbf{u}(k+1) \\ \mathbf{y}(k) &= C \otimes \mathbf{x}(k), \end{aligned} \tag{6}$$

for $k = 1,2,3, \ldots$, with the initial condition $\mathbf{x}(0) = \mathbf{x}_0$, $A \in \mathbb{R}^{n}_{\max}$, $B \in \mathbb{R}^{n\times m}_{\max}$, and $C \in \mathbb{R}^{I\times n}_{\max}$. The vector $\mathbf{x}(k) \in \mathbb{R}^{n}_{\max}$ expresses the state, $\mathbf{u}(k) \in \mathbb{R}^{m}_{\max}$ is the input vector, and $\mathbf{y}(k) \in \mathbb{R}^{I}_{\max}$ is the output vector of the system at the time k.

SLMI as in the above definition will be briefly written with SLMI (A, B, C) and written with SLMI $(A, B, C, \mathbf{x}_0)$, if the initial condition $\mathbf{x}(0) = \mathbf{x}_0$ is given, SLMI with one input and one output will be called SLMI SISO. Whereas SLMI with more than one *input* and more than one *output* will be called SLMI MIMO.

Time-Invariant Max-Plus Linear System Input-Output Analysis

If the initial condition and an SLMI input-output line . If the initial condition and an SLMI input-output line . If the initial condition and an input row are given for an SLMI$(A, B, C, \mathbf{x}_0)$, then a system state vector row and a system output row can be recursively determined .

Suppose the initial condition $\boldsymbol{x}(0) = [0,1,\varepsilon]^T$ of the system means that the processing unit P_1 and P_2 consecutively start its activity at time 0 and 1 while the processing unit P_3 is still empty and has to wait for the arrival of input from P_1 and P_2. The raw material is entered the system at the time

0,9,12,24 and so on which means given an input line, $u(1)=0, u(2)=9, u(3)=12, u(4)=24$ and so on, with $u(k) \leq u(k+1)$ for $k=1,2,3,\dots$ each recursively defined line vector of the following state:

$$\boldsymbol{x}(1) = A \otimes \boldsymbol{x}(0) \oplus B \otimes u(1) = \begin{bmatrix} 5 & \varepsilon & \varepsilon \\ \varepsilon & 6 & \varepsilon \\ 11 & 12 & 3 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \\ \varepsilon \end{bmatrix} \oplus \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} \otimes 0$$

$$= \begin{bmatrix} 5 \\ 7 \\ 13 \end{bmatrix} \oplus \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 13 \end{bmatrix}$$

$$\boldsymbol{x}(2) = A \otimes \boldsymbol{x}(1) \oplus B \otimes u(2) = \begin{bmatrix} 10 \\ 13 \\ 19 \end{bmatrix} \oplus \begin{bmatrix} 11 \\ 9 \\ 17 \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 19 \end{bmatrix}$$

$$\boldsymbol{x}(3) = A \otimes \boldsymbol{x}(2) \oplus B \otimes u(3) = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix} \oplus \begin{bmatrix} 14 \\ 12 \\ 20 \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$\boldsymbol{x}(4) = A \otimes \boldsymbol{x}(3) \oplus B \otimes u(4) = \begin{bmatrix} 21 \\ 25 \\ 31 \end{bmatrix} \oplus \begin{bmatrix} 26 \\ 24 \\ 32 \end{bmatrix} = \begin{bmatrix} 26 \\ 25 \\ 32 \end{bmatrix}, \text{ and so on.}$$

Then the system output line is obtained as follows by using $y(k) = x_3(k) + 3; y(1) = 16, y(2) = 22, y(3) = 28, y(4) = 35$, and so on which means the product will be able to be retrieved at time 16,22,28,35 and so on.

3. Results and Discussion

Based on the results of interviews and questionnaires about the production of potato donuts, it is known that the process and time for the production of Naziza potato donuts are known as the following table:

Table 1. Production Time of Naziza's Potato Donuts

No	Code	Process	Time (minutes)
1	t_1	Preparation of boiled potatoes	5
2	P_1	Potatoes in Boil	20
3	t_2	Preparation of containers for dough	1
4	P_2	Pour flour and sugar into the flour container	3
5	t_3	Preparation for mashing potatoes	2
6	P_3	Mashed potatoes	5
7	t_4	Preparation of yeast to add flour dough	1
8	P_4	Add yeast to flour dough	1
9	t_5	Transfer of the already finely cooked potatoes into the container	2

10	t_6	Preparation of flour dough to add the already smooth potatoes	2
11	P_5	Add the potatoes to the flour mixture	2
12	t_7	preparation of ingredients for putting into potato dough	1
13	P_6	Add egg yolks, milk, margarine, salt to the potato batter	3
14	t_8	Preparation of stirring potato dough	2
15	P_7	Stir the potato dough	20
16	t_9	Transfer of potato dough from container to paperwork	4
17	P_8	Divide the potato dough into rounds then cover until fluffy	60
18	t_{10}	preparation of dough to be shaped into a donut shape	10
19	P_9	Shape the dough like a donut and cover it again until fluffy	60
20	t_{11}	Preparing a pan for frying donuts	3
21	P_{10}	Fried Donuts	20
22	t_{12}	Preparing toppings for donuts	5
23	P_{11}	Donuts Adorable Top	17

The Potato Donut production system has eleven process units, which $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}$ dan P_{11} have the corresponding time interval in Table 1. In the Potato Donut production system, the raw materials are divided into two processes, namely the process of boiling potatoes and making dough. Based on the results of the research, the production system of Potato Donuts can be presented in the following chart:

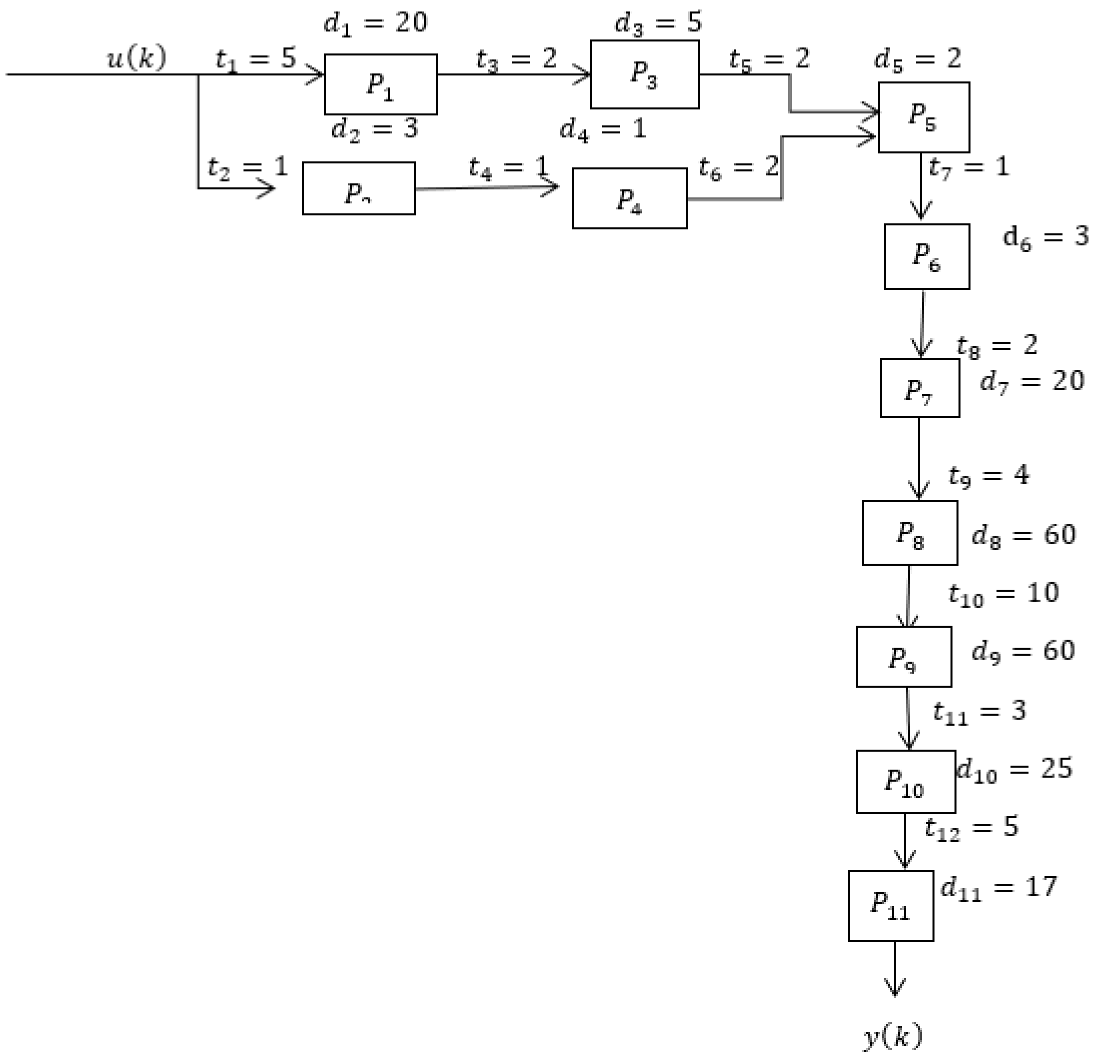

Figure 1. Potato Donut's Production Modeling Chart

Description:

P_i = process i ,i=1,2,3,4,.......,11

t_i = Preparation of the first process from the previous process

d_i = time used for P_i

Modeling of the Max-Plus Time Invariant (SLMI) Algebraic Linear System of Potato Donut Production

This potato donut's production system consists of 11 processing units, which $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}$ dan P_{11} processing time (every minutes) are successively. A unit of time (minutes) is given $d_1 = 20, d_2 = 3, d_3 = 5, d_4 = 1, d_5 = 2, d_6 = 3, d_7 = 20,\ d_8 = 60, d_9 = 60,\ d_{10} = 25, d_{11} = 17$.

It is assumed that each processing unit starts working with an available raw material. Difined :

- $u(k+1)$: The time when raw materials are fed into the system for processing ke-$(k+1)$
- $x_i(k)$: The time when the i-i processing unit starts working for processing ke $-\ k$
- $y(k)$: The time when the *completed production leaves* the system.
- A : The time of the ongoing potato donut production process.
- B : Transfer time from the beginning of the raw material entry into the system Production before the first event
- C : Final event time and transfer time before donut production potatoes can be picked up or finished working.

So that from the production chart using the Max-Plus Algebra operation , the following is obtained:

$$x_1(k+1) = \text{maks}\left(20 + x_1(k), 5 + u(k+1)\right)$$

$$x_2(k+1) = \text{maks}\left(3 + x_2(k), 1 + u(k+1)\right)$$

$$x_3(k+1) = \text{maks}\ (42 + x_1(k), 5 + x_3(k), 27 + u(k+1))$$

$$x_4(k+1) = \text{maks}\left(7 + x_2(k), 1 + x_4(k), 5 + u(k+1)\right)$$

$$x_5(k+1) = \text{maks}\ (49 + x_1(k), 10 + x_2(k), 12 + x_3(k), 4 + x_4(k), 2 + x_5(k), 34 + u(k+1))$$

$$x_6(k+1) = \text{maks}(52 + x_1(k), 13 + x_2(k), 15 + x_3(k), 7 + x_4(k), 5 + x_5(k), 3 + x_6(k), 37 + u(k+1)$$

$$x_7(k+1) = \text{maks}(57 + x_1(k), 18 + x_2(k), 20 + x_3(k), 12 + x_4(k), 10 + x_5(k),$$
$$8 + x_6(k), 20 + x_7(k), 42 + u(k+1))$$

$$x_8(k+1) = \text{maks}\ (81 + x_1(k), 42 + x_2(k), 44 + x_3(k), 36 + x_4(k), 34 + x_5(k), 32 + x_6(k),$$
$$44 + x_7(k), 60 + x_8(k), 66 + u(k+1))$$

$$x_9(k+1) = \text{maks}(151 + x_1(k), 112 + x_2(k), 114 + x_3(k), 106 + x_4(k),$$
$$104 + x_5(k), 102 + x_6(k), 114 + x_7(k), 130 + x_8(k), 60 + x_9(k),$$
$$136 + u(k+1))$$

$$x_{10}(k+1) = \text{maks}(\ 214 + x_1(k), 175 + x_2(k), 177 + x_3(k), 169 + x_4(k),$$
$$167 + x_5(k), 165 + x_6(k), 177 + x_7(k), 193 + x_8(k),$$
$$123 + x_9(k), 25 + x_{10}(k), 199 + u(k+1))$$

$$x_{11}(k+1) = \text{maks}\ (244 + x_1(k), 205 + x_2(k), 207 + x_3(k), 199 + x_4(k),$$
$$197 + x_5(k), 195 + x_6(k), 207 + x_7(k), 223 + x_8(k),$$
$$153 + x_9(k), 55 + x_{10}(k), 17 + x_{11}(k), 229 + u(k+1)) \quad (7)$$

Equations (7) can we write into the Max-Plus Linear Algebra system

$$x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1),$$
$$y(k) = C \otimes x(k) \quad (8)$$

with

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \\ x_9(k) \\ x_{10}(k) \\ x_{11}(k) \end{bmatrix}, x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \\ x_7(k+1) \\ x_8(k+1) \\ x_9(k+1) \\ x_{10}(k+1) \\ x_{11}(k+1) \end{bmatrix}$$

$$A = \begin{bmatrix} 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 42 & \varepsilon & 5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 7 & \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 49 & 10 & 12 & 4 & 2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 52 & 13 & 15 & 7 & 5 & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 57 & 18 & 20 & 12 & 10 & 8 & 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 81 & 42 & 44 & 36 & 34 & 32 & 44 & 60 & \varepsilon & \varepsilon & \varepsilon \\ 151 & 112 & 114 & 106 & 104 & 102 & 114 & 130 & 60 & \varepsilon & \varepsilon \\ 214 & 175 & 177 & 169 & 167 & 165 & 177 & 193 & 123 & 25 & \varepsilon \\ 244 & 205 & 207 & 199 & 197 & 195 & 207 & 223 & 153 & 55 & 17 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 \\ 1 \\ 27 \\ 5 \\ 34 \\ 37 \\ 42 \\ 66 \\ 136 \\ 199 \\ 229 \end{bmatrix}, \; C = [\varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; 17]$$

The initial conditions are given as follows $x(0) = [0 \; 1 \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \; \varepsilon \;]^T$ and

input lines $u = [0 \; 20 \; 40 \; 60 \; 80 \; 100 \; 120 \; 140 \; 160 \; 180 \; 200]^T$.

SLMI Input - Output Analysis in Optimizing The Production Time of Potato Donuts

Based on the calculation of the input and output time of the potato donut production process is presented in Table 2 as follows:

Table 2. Calculation of input state time and output process length Naziza potato donut production

U(k)	1	2	3	4	5	6	7
x_1	20	40	60	80	100	120	140
x_2	4	7	10	13	16	19	21
x_3	42	62	82	102	122	142	162
x_4	8	11	14	17	20	23	26
x_5	49	69	89	109	129	149	169
x_6	52	72	92	112	132	152	172
x_7	57	77	97	117	137	157	177
x_8	81	141	201	261	321	381	441
x_9	151	211	271	331	391	451	511
x_{10}	214	274	334	394	454	514	574
x_{11}	244	304	364	424	484	544	604
$y(k)$	261	321	381	441	501	561	621

The time obtained to maximize production in a day is approximately 9 hours (540 minutes) with working time starting from 08.00-17.00 WIB presented in Table 4.1.1 with a production completion time of 540 minutes shows that in a day the potato donut business can only do a few productions due to time constraints and this also means that the number of potato donuts that can be ordered is limited in quantity. The following is a periodic schedule of potato donut production time:

Table 3. Periodic Schedule of Production Time of Naziza’s Potato Donuts

Stages of making Potato Donuts	Production Sequence					
	1	2	3	4	5	6
Boiled Potatoes	08.05	08.25	08.45	09.10	09.25	09.45
Pour flour and sugar into a container	08.06	08.09	08.12	08.15	08.18	08.21
mashed potatoes	08.27	08.47	09.07	09.27	09.47	10.07
Add yeast to flour dough	08.10	08.13	08.16	08.19	08.22	08.25
Mix the potatoes into the flour mixture	08.33	08.54	09.14	09.34	08.54	10.14
Add egg yolks, milk, margarine, salt to the potato mixture	08.36	08.57	09.17	09.37	09.57	10.17
Stir in the potato dough	08.41	08.62	09.22	09.42	10.02	10.22

Stir the dough into a round shape then cover until fluffy	09.05	09.26	11.26	12.26	13.26	14.26
Shape the dough like a donut and cover again until fluffy	10.15	10.36	11.36	12.36	13.36	14.36
Fried Donuts	11.18	12.39	13.39	14.39	15.40	16.40
Donuts Adorable Top	11.48	13.09	14.09	15.09	16.09	17.10
Donut intake	12.26	13.26	14.26	15.26	16.26	17.26

The output results above with the input time required when the material is entered to be processed from the time of the first production to the 5th production, which meets for its daily activities within the predetermined working time from 08.00-17.00 WIB, namely in Table 4.3.2, it shows that when the production of Potato Donuts is carried out optimally and continuously, the daily results of potato donuts can reach optimal production 5th.

Naziza Potato Donut's owner can determine the optimal time to start production of potato donuts in order to meet the demand of consumers who have placed orders for potato donuts before the production process begins. This problem can be solved by using *Max-Plus* algebra optimization. By determining the optimal pick-up time, it is determined using the following formula :

$$y = K \otimes x_0 \oplus H \otimes u \tag{9}$$

with

$$K = \begin{bmatrix} C \otimes A \\ C \otimes A^{\otimes 2} \\ \vdots \\ C \otimes A^{\otimes p} \end{bmatrix} \text{ and } H = \begin{bmatrix} C \otimes B & \varepsilon & \cdots & \varepsilon \\ C \otimes A \otimes B & C \otimes B & \cdots & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ C \otimes A^{\otimes p-1} \otimes B & C \otimes A^{\otimes p-2} \otimes B & \cdots & C \otimes B \end{bmatrix}.$$

From the results of the Matlab calculations, it was obtained that:

$$y = [306; 366; 426; 486; 546; 606; 666; 726; 786; 846; 906]^T.$$

Then producers can determine the optimal time (largest sub-finish) to start production activities using the formula : $\hat{u} = H^T \otimes (-y)$,

$$\hat{y} = H \otimes \hat{u},$$
$$\tilde{u} = \hat{u} \otimes \frac{\delta}{2},$$
$$\tilde{y} = H \otimes \tilde{u},$$

where:

$\hat{u}$ = Fastest time to start production.

$\hat{y}$ = Fastest production finish time.

$\tilde{u}$ = longest time to start production.

$\tilde{y}$ = longest production finish time.

The *output value* is obtained from the Matlab calculation which can be seen as below:

$\hat{u} = [60; 120; 180; 240; 300; 360; 420; 480; 540; 600; 660]^T$.

$\hat{y} = [306; 366; 426; 486; 546; 606; 666; 726; 786; 846; 906]^T$.

$\tilde{u} = [63; 123; 183; 243; 303; 363; 423; 483; 543; 603; 663]^T$.

$\tilde{y} = [309; 369; 429; 489; 549; 609; 669; 729; 789; 849; 909]^T$.

This calculation makes it easier for producers to determine the optimal time to start the potato donut production process. This u and $\tilde{u}$ are the largest sub-completion as well as optimal time in the production system. u and $\tilde{u}$ are used to determine the periodic production schedule so that the time from starting to insert the material until the time of picking up the product within a period of one day (WIB) can be seen in Table 4.

Table 4. Naziza's Potato Donut Order Schedule

Order to-	$(\hat{u})$	$(\hat{y})$	$(\tilde{u})$	$(\tilde{y})$
1	08.00	13.06	08.03	13.09
2	09.00	14.06	09.03	14.09
3	10.00	15.06	10.03	15.09
4	11.00	16.06	11.03	16.09
5	12.00	17.06	12.03	17.09
6	13.00	18.06	13.03	18.09

The optimal time for producers to start the production process from Table 4 for the first order is by choosing $\tilde{u}$ because by starting production at 08.00 then consumers who have ordered potato donuts can take their order at 13.06 or later. From this, the producer can serve consumers on time by using the table as a reference to start production. In addition, producers can also fulfill orders 2 and 3 on time by choosing $\hat{u}$ as the optimal time (largest sub-finish). The fifth production and so on cannot be used as a reference because it has passed the working time in the reference because it has passed the working time in production, namely 08.00-17.00 WIB. The scheduling that is carried out as shown in Table 4 is a scheduling used by producers to optimize input time (entering ingredients) and output time (completing production time) so that the table can be used as one of the references to start production of potato donuts can be optimized so that potato donut's orders for consumers can be served on time.

4. Conclusion

From this study, the Linear Method Max-Plus Time Invariant one Input one Output (SLMI SISO) in the potato donut's production system can be concluded that :

1. The modeling of the Max-Plus Time Invariant (SLMI) *Algebraic Linear System* of Naziza potato donut production is as follows:

$$A = \begin{bmatrix} 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 42 & \varepsilon & 5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 7 & \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 49 & 10 & 12 & 4 & 2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 52 & 13 & 15 & 7 & 5 & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 57 & 18 & 20 & 12 & 10 & 8 & 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 81 & 42 & 44 & 36 & 34 & 32 & 44 & 60 & \varepsilon & \varepsilon & \varepsilon \\ 151 & 112 & 114 & 106 & 104 & 102 & 114 & 130 & 60 & \varepsilon & \varepsilon \\ 214 & 175 & 177 & 169 & 167 & 165 & 177 & 193 & 123 & 25 & \varepsilon \\ 244 & 205 & 207 & 199 & 197 & 195 & 207 & 223 & 153 & 55 & 17 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 \\ 1 \\ 27 \\ 5 \\ 34 \\ 37 \\ 42 \\ 66 \\ 136 \\ 199 \\ 229 \end{bmatrix}, \quad C = [\varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad 17]$$

   The initial conditions are given as follows: $x(0) = [0\ 1\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ ]^T$

   and input lines $u = [0\ 20\ 40\ 60\ 80\ 100\ 120\ 140\ 160\ 180\ 200]^T$.

2. There are 2 ways to optimize the production time of potato donuts with the Max-Plus Linear System Method Invariant Time (SLMI), namely for producers to determine the production start time by choosing between $\hat{u}$ or $\tilde{u}$ so that completion time of the product $\hat{y}$ or $\tilde{y}$ that is close to the order pick-up time that has been determined by the consumer. So, producers can choose $\hat{u}$ or $\tilde{u}$ (SLMI's largest sub-settlement in this production system) in order to optimize the production time of potato donuts so that production results can meet consumer demand and potato donut orders can also be served on time.

References

Carnia, E., Wilopo, R., Napitupulu, H., Anggriani, N., & Supriatna, A. K. (2023). Modified Kleene Star Algorithm Using Max-Plus Algebra and Its Application in the Railroad Scheduling Graphical User Interface. *Computation*, *11*(1).

Fatimah, M. F., & Darmawati, D. (2023). Near-Modul Kuat Faktor. *Jambura Journal of Mathematics*, *5*(1), 210–217.

Lestari, F. D. W. I., Matematika, P. S., Sains, J., Sains, F., Teknologi, D. A. N., Islam, U., Sunan, N., & Surabaya, A. (2018). *K-ISOMORFISME DALAM K-ALJABAR*.

Manambing, A., Mandey, S., & Tielung, M. V. . (2018). Analisis Pengaruh Orientasi Pasar Dan Keunggulan Bersaing Terhadap Kinerja Pemasaran (Studi Kasus Umkm Kuliner Tinutuan Di Manado). *Jurnal EMBA: Jurnal Riset Ekonomi, Manajemen, Bisnis Dan Akuntansi*, *6*(4), 3803–3812.

Rambow, T., & Kiencke, U. (2005). Max-Plus-Algebra, Teil 1 (Max-Plus Algebra, Part 1). *Auto*, *53*(10), A1–A4.

Rudhito, M. A. (2011). *A-11 Sistem Linear Max-Plus Interval Waktu Invariant*. *1996*.

Rudhito, M. A. (2016a). Aljabar max-plus dan penerapannya. *Universitas Sanata Dharma Yogyakarta*.

Rudhito, M. A. (2016b). *ALJABAR MAX-PLUS DAN PENERAPANNYA*.

Rudhito, M. A., Wahyuni, S., Suparwanto, A., & Susilo, F. (2012). Matriks atas Aljabar Max-Plus Interval. *Jurnal Natur Indonesia*, *13*(2), 94.

Schutter, B. de. (n.d.). *Max-algebraic system theory for discrete event systems*.

Subiono. (2015). *Aljabar Min-Max Plus dan Terapannya*. 173.

Vinatra, S. (2023). Peran Usaha Mikro, Kecil, dan Menengah (UMKM) dalam Kesejahteraan Perekonomian Negara dan Masyarakat. *Jurnal Akuntan Publik*, *1*(3), 1–08.